$A \cup B$. In Problems 99–104, write the elements of each set, using sets A, B, and C below.

$A = \{7, 8, 9, 10, 11, 12\}$ $B = \{10, 11, 12, 13, 14, 15\}$
$C = \{11, 12, 13, 14, 15\}$

99. $A \cup B$

100. $A \cap B$

101. $B \cup C$

102. $B \cap C$

103. $A \cap C$

104. $A \cup C$

105. If $A = \{\text{even integers}\}$ and $B = \{\text{whole numbers less than } 11\}$, find $A \cap B$.

106. If $X = \{48, 49, 50, \ldots\}$ and $Y = \{60, 62, 64, \ldots, 80\}$, find $X \cap Y$.

107. When writing subsets, it is important to be orderly when creating the list. Think of a pattern and then answer the following:

(a) List all possible subsets of set Z where $Z = \{1, 2, 3, 4\}$. *Hint:* The empty set is a subset of every set.

(b) How many subsets did you find?

108. Use the set $M = \{a, b, c\}$ to answer the following:

(a) List all possible subsets of M. *Hint:* The empty set is a subset of every set.

(b) How many subsets did you find?

(c) Determine a rule for finding the number of subsets of a set that has n elements.

Explaining the Concepts

109. Write a definition of "rational number" in your own words. Describe the characteristics to look for when deciding whether a number is in this set.

110. Write a definition of "irrational number" in your own words. Describe the characteristics to look for when deciding whether a number is in this set.

1.4 Adding, Subtracting, Multiplying, and Dividing Integers

Objectives

1. Add Integers
2. Determine the Additive Inverse of a Number
3. Subtract Integers
4. Multiply Integers
5. Divide Integers

Are You Ready for This Section?

Before getting started, take this readiness quiz. If you get a problem wrong, go back to the section cited and review the material.

R1. Write $\frac{16}{36}$ as a fraction in lowest terms. [Section 1.2, pp. 12–13]

R2. $|-5| = \_\_$. [Section 1.3, pp. 24–25]

In this section, we perform addition, subtraction, multiplication, and division, called **operations,** on integers. The symbols used in algebra for these operations are $+, -, \cdot$, and /, respectively. The results of these four operations are called the **sum, difference, product**, and **quotient,** respectively. Table 2 summarizes these ideas.

Table 2

Operation	Symbols	Words
Addition	$a + b$	Sum: *a* plus *b*
Subtraction	$a - b$	Difference: *a* minus *b*
Multiplication	$a \cdot b, (a) \cdot b, a \cdot (b), (a) \cdot (b), ab, (a)b, a(b), (a)(b)$	Product: *a* times *b*
Division	a/b or $\frac{a}{b}$	Quotient: *a* divided by *b*

Ready?...Answers **R1.** $\frac{4}{9}$ **R2.** 5

In algebra, we avoid using the multiplication sign $\times$ used in arithmetic. Instead, we multiply two expressions that are placed next to each other without an operation symbol, as in ab or that are in parentheses, as in $(a)(b)$, or we use $\cdot$ as in $a \cdot b$.

A **mixed number** is a whole number followed by a fraction. We do not use mixed numbers in algebra. When you see a mixed number, rewrite it as a fraction. Recall that to write $3\frac{2}{5}$ as a fraction, we multiply the whole number 3 by the denominator 5, obtaining 15, and then add this result to the numerator 2 to get 17. This result is the numerator of the fraction. The denominator remains 5. Thus

$$3\frac{2}{5} = \frac{17}{5} \leftarrow 3 \cdot 5 + 2$$

In algebra, mixed numbers are confusing because the lack of an operation symbol between two terms means multiplication. To avoid confusion, write $3\frac{2}{5}$ as 3.4 or as $\frac{17}{5}$.

Work Smart

Do not use mixed numbers in algebra.

1 Add Integers

Adding Integers with the Same Sign Using a Number Line

We will use a real number line to discover a pattern for adding integers. When we add a positive integer, we move to the right on the number line, and when we add a negative integer, we move to the left on the number line.

Remember, the *sign* of a number indicates whether the number is positive or negative. For example, the sign of 4 is positive, while the sign of -12 is negative. We will first consider adding integers with the same sign.

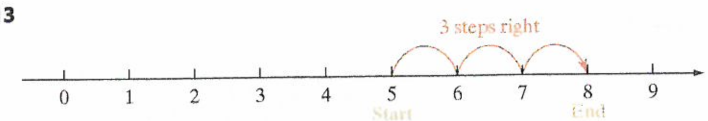
EXAMPLE 1 **Adding Two Positive Integers Using a Number Line**

Find the sum: $5 + 3$

Solution

We begin at 5 on the number line and move 3 spaces to the right, so $5 + 3 = 8$. See Figure 13.

Figure 13



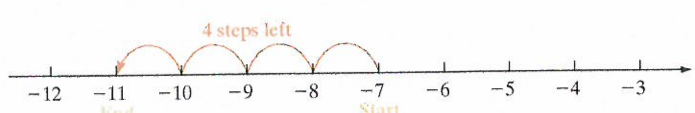
EXAMPLE 2 **Adding Two Negative Integers Using a Number Line**

Find the sum: $-7 + (-4)$

Solution

We begin at -7 on the number line and move 4 spaces to the left, so $-7 + (-4) = -11$. See Figure 14.

Figure 14



Quick ✓

1. The answer to an addition problem is called the ____

In Problems 2 and 3, use a number line to find each sum.

2. $8 + 6$ 3. $-3 + (-5)$

Adding Integers with Different Signs Using a Number Line

We now consider the sum of two integers with different signs.

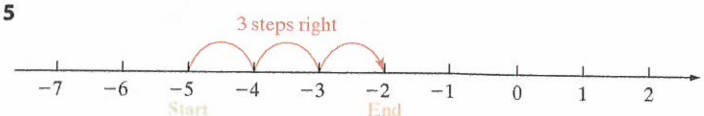
EXAMPLE 3 Adding Integers with Different Signs Using a Number Line

Find the sum: $-5 + 3$

Solution

We begin at -5 and then move 3 units to the right. From Figure 15 we see that $-5 + 3 = -2$.

Figure 15



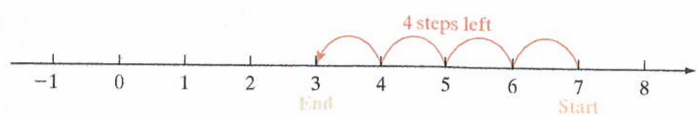
EXAMPLE 4 Adding Integers with Different Signs Using a Number Line

Find the sum: $7 + (-4)$

Solution

We begin at 7 and move 4 spaces to the left. We see that $7 + (-4) = 3$. See Figure 16.

Figure 16



Quick ✓

In Problems 4–7, use a number line to find each sum.

4. $-1 + 4$ 5. $3 + (-4)$ 6. $-8 + 4$ 7. $17 + (-3)$

Adding Integers Using Absolute Value

Did you discover a pattern for adding integers from Examples 1–4? To add integers with the same sign (both positive or both negative), add the absolute values of the integers and attach the common sign. To add integers with different signs (one positive and one negative), subtract the smaller absolute value from the larger absolute value and attach the sign of the integer having the larger absolute value.

EXAMPLE 5 How to Add Integers with the Same Sign Using Absolute Value

Find $-16 + (-24)$ using absolute value.

Step-by-Step Solution

Step 1: Add the absolute values of the two integers.

We have $|-16| = 16$ and $|-24| = 24$. So

$$16 + 24 = 40$$

Step 2: Attach the common sign, either positive or negative.

Both integers are negative in the original problem, so

$$-16 + (-24) = -40$$

EXAMPLE 6 How to Add Integers with Different Signs Using Absolute Value

Find $-31 + 16$ using absolute value.

Step-by-Step Solution

Step 1: Subtract the smaller absolute value from the larger absolute value.

We have $|-31| = 31$ and $|16| = 16$. The smaller absolute value is 16, so we compute

$$31 - 16 = 15$$

Step 2: Attach the sign of the integer with the larger absolute value.

The larger absolute value is 31, which was a negative number in the original problem, so the sum is negative. Therefore,

$$-31 + 16 = -15$$

Adding Two Integers Using Absolute Value

To add integers with the same sign (both positive or both negative),

Step 1: Add the absolute values of the two integers.

Step 2: Attach the common sign.

To add integers with different signs (one positive and one negative),

Step 1: Subtract the smaller absolute value from the larger absolute value.

Step 2: Attach the sign of the integer with the larger absolute value.

Quick ✓

8. The sum of two negative integers will be ______.

In Problems 9–12, use absolute value to find each sum.

9. $-11 + 7$

10. $5 + (-8)$

11. $-8 + (-16)$

12. $-94 + 38$

2 Determine the Additive Inverse of a Number

What is $3 + (-3)$? What is $10 + (-10)$? What is $-143 + 143$? The answer to all three of these questions is 0. These results are true in general.

Work Smart

The additive inverse of a, $-a$ should not be called the *negative* of a because it suggests that the opposite is a negative number, which may not be true! For example, the additive inverse of -11 is 11, a positive number.

Additive Inverse Property

For any real number a other than 0, there is a real number $-a$, called the **additive inverse,** or **opposite,** of a, having the following property.

$$a + (-a) = -a + a = 0$$

Any two numbers whose sum is zero are additive inverses, or opposites, of each other.

EXAMPLE 7 Finding an Additive Inverse

(a) The additive inverse of 9 is -9 because $9 + (-9) = 0$.

(b) The additive inverse of -12 is $-(-12) = 12$ because $-12 + 12 = 0$.

Notice from Example 7(b) that $-(-a) = a$ for any real number a.

Quick ✓

13. For any real number a other than 0, there is a real number $-a$, called the ______ ______, or ______, of a such that $a + (-a) = -a + a = \_$.

In Problems 14–17, determine the additive inverse of the given real number.

14. 7 **15.** -21 **16.** $-\frac{8}{5}$ **17.** 5.75

3 Subtract Integers

To subtract integers, we rewrite the subtraction problem as an addition problem and use our addition rules.

From arithmetic, we write $10 - 6 = 4$. Using the additive inverse, we can write $10 - 6 = 4$ as the addition problem $10 + (-6) = 4$.

In Words

To subtract b from a, add the opposite of b to a.

Definition

The **difference** $a - b$, read "a minus b" or "a less b," is defined as

$$a - b = a + (-b)$$

EXAMPLE 8 How to Subtract Integers

Compute the difference: $-18 - (-40)$

Step-by-Step Solution

Step 1: Change the subtraction problem to an equivalent addition problem. $-18 - (-40) = -18 + 40$

Step 2: Find the sum. $= 22$

In Words

The problem in Example 8 is read "negative eighteen minus negative 40" not "negative 18 minus minus 40."

Subtracting Nonzero Integers

1. Change the subtraction problem to an equivalent addition problem using $a - b = a + (-b)$.
2. Find the sum.

Quick ✓

18. The answer to a subtraction problem is called the ______.

19. The subtraction problem $-3 - 10$ is equivalent to $-3 +$ ____

In Problems 20–22, find the value of each expression.

20. $59 - (-21)$ **21.** $-32 - 146$ **22.** $-19 - (-40)$

For the remainder of this text, to **evaluate** will mean to find the numerical value of an expression. To evaluate an expression that has both addition and subtraction, change all subtraction to addition. Then add from left to right.

EXAMPLE 9 Evaluating an Expression with Three Integers

Evaluate: $10 - 18 + 25$

Solution

Work Smart

When adding and subtracting more than two numbers, add in order from left to right.

$$10 - 18 + 25 = 10 + (-18) + 25$$

Add from left to right: $= -8 + 25$

$= 17$

EXAMPLE 10 **Evaluating an Expression with Four Integers**

Evaluate: $162 - (-46) + 80 - 274$

Solution

$$\begin{aligned} 162 - (-46) + 80 - 274 &= 162 + 46 + 80 + (-274) \\ \text{Add from left to right: } &= 208 + 80 + (-274) \\ &= 288 + (-274) \\ &= 14 \end{aligned}$$

Quick ✓

In Problems 23–26, evaluate each expression.

23. $8 - 13 + 5$

24. $-27 - 49 + 18$

25. $3 - (-14) - 8 + 3$

26. $-825 + 375 - (-735) + 265$

4 Multiply Integers

In the statement $9 \cdot 2 = 18$, 9 and 2 are the **factors** and 18 is the **product.**

$$\underbrace{9}_{\text{factor}} \cdot \underbrace{2}_{\text{factor}} = \underbrace{18}_{\text{product}}$$

Recall that we can think of multiplication as repeated addition.

$$3 \cdot 4 = \underbrace{4 + 4 + 4}_{\text{Add 4 three times}} = 12$$

Notice that the product of two positive factors is positive. We knew that from arithmetic! But what is $3 \cdot (-4)$?

$$3 \cdot (-4) = \underbrace{-4 + (-4) + (-4)}_{\text{Add } -4 \text{ three times}} = -12$$

We conclude that the product of two real numbers with *different signs* is negative.

What about the product of two negative numbers? Consider the pattern below.

$$\begin{aligned} -4 \cdot 3 &= -12 \\ -4 \cdot 2 &= -8 \\ -4 \cdot 1 &= -4 \\ -4 \cdot 0 &= 0 \end{aligned}$$

Each time the second factor decreases by 1, the product increases by 4. Assuming this pattern continues, we would have

$$\begin{aligned} -4 \cdot -1 &= 4 \\ -4 \cdot -2 &= 8 \end{aligned}$$

The pattern suggests that the product of two negative numbers is a positive number.

In Words

The product is *positive* if the signs of the two factors are the *same* and *negative* if the signs of the two factors are *different*.

Rules of Signs for Multiplying Two Integers

1. The product of two positive integers is positive.
2. The product of one positive integer and one negative integer is negative.
3. The product of two negative integers is positive.

EXAMPLE 11 **Multiplying Integers**

(a) $2(-4) = -8$ **(b)** $-6(5) = -30$

(c) $(-7)(-8) = 56$ **(d)** $-25(18) = -450$

Quick ✓

27. The product of two integers with the same sign is ______.

In Problems 28–32, find the product.

28. $-3(7)$ **29.** $13(-4)$ **30.** $5 \cdot 16$

31. $-9(-12)$ **32.** $(-13)(-25)$

Find the Product of Several Integers

To find the product of more than two integers, multiply in order, from left to right.

EXAMPLE 12 **Multiplying Three or More Integers**

Find the product:

(a) $3 \cdot (-4) \cdot (-7)$ **(b)** $-8 \cdot (-1) \cdot 4 \cdot (-5)$

Solution

To find each product, multiply from left to right.

(a) $3 \cdot (-4) \cdot (-7) = -12 \cdot (-7)$
$= 84$

(b) $-8 \cdot (-1) \cdot 4 \cdot (-5) = 8 \cdot 4 \cdot (-5)$
$= 32 \cdot (-5)$
$= -160$

Work Smart

If we multiply an *even* number of negative factors, the product is *positive*.

If we multiply an *odd* number of negative factors, the product is *negative*.

Quick ✓

33. *True or False* The product of thirteen negative factors is negative.

In Problems 34 and 35, find the product.

34. $-3 \cdot 9 \cdot (-4)$ **35.** $3 \cdot (-4) \cdot (-5) \cdot (-6)$

5 Divide Integers

When we divide, the numerator is the **dividend,** the denominator is the **divisor,** and the answer is the **quotient.**

$$\text{dividend} \rightarrow \frac{28}{7} = 4 \leftarrow \text{quotient} \quad \text{or} \quad \underbrace{28}_{\text{dividend}} \div \underbrace{7}_{\text{divisor}} = \underbrace{4}_{\text{quotient}}$$
(divisor → 7)

To discuss division of integers, we need to introduce the idea of a *multiplicative inverse.*

In Words

Any two numbers whose product is 1 are called multiplicative inverses, or reciprocals, of each other.

Multiplicative Inverse (Reciprocal) Property

For each *nonzero* real number a, there is a real number $\frac{1}{a}$, called the **multiplicative inverse,** or **reciprocal,** of a, having the following property:

$$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1 \quad \text{where } a \neq 0$$

EXAMPLE 13 Finding the Multiplicative Inverse (or Reciprocal) of an Integer

(a) The multiplicative inverse, or reciprocal, of 5 is $\frac{1}{5}$.

(b) The multiplicative inverse, or reciprocal, of -8 is $-\frac{1}{8}$.

Quick ✓

In Problems 36 and 37, find the multiplicative inverse, or reciprocal, of each integer.

36. 6 **37.** -2

Now we can define division of integers.

Definition

If b is a nonzero integer, the **quotient** $\frac{a}{b}$, read as "a divided by b" or "the **ratio** of a to b," is defined as

$$\frac{a}{b} = a \cdot \frac{1}{b} \quad \text{if} \quad b \neq 0$$

For example, $\frac{40}{8} = 40 \cdot \frac{1}{8}$ and $\frac{12}{7} = 12 \cdot \frac{1}{7}$. Because division can be represented as multiplication, the same rules of signs that apply to multiplication also apply to division.

In Words

These are the same rules we saw in multiplying integers. A positive divided by a positive is positive, a positive divided by a negative is negative, and so on.

Rules of Signs for Dividing Two Integers

1. The quotient of two positive integers is positive. That is, $\frac{+a}{+b} = \frac{a}{b}$.
2. The quotient of one positive integer and one negative integer is negative. That is, $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$.
3. The quotient of two negative integers is positive. That is, $\frac{-a}{-b} = \frac{a}{b}$.

Finding the quotient of two integers is the same as writing a fraction in lowest terms.

EXAMPLE 14 Finding the Quotient of Two Integers

Find each quotient:

(a) $\frac{-90}{20}$ **(b)** $200 \div (-5)$

Solution

(a)

$$\frac{-90}{20} = \frac{-9 \cdot 10}{2 \cdot 10}$$

Divide out the common factor:

$$= \frac{-9 \cdot \cancel{10}}{2 \cdot \cancel{10}}$$

$$= \frac{-9}{2}$$

$\frac{-a}{b} = -\frac{a}{b}$:

$$= -\frac{9}{2}$$

(b)

$$200 \div (-5) = \frac{200}{-5}$$
$$= \frac{40 \cdot 5}{-1 \cdot 5}$$
Divide out the common factor: $$= \frac{40 \cdot \cancel{5}}{-1 \cdot \cancel{5}}$$
$$= -40$$

Work Smart: Study Skills

Selected problems in the exercise sets are identified by green color. For extra help, worked solutions to these problems are available in MyMathLab.

Quick ✓

38. In the division problem $\frac{18}{3} = 6$, 18 is called the _______, 3 is called the _______, and 6 is called the _______.

39. *True or False:* The quotient of two negative numbers is positive.

In Problems 40–42, find the quotient.

40. $\frac{20}{-4}$ **41.** $\frac{-72}{20}$ **42.** $-63 \div -7$

1.4 Exercises

MyMathLab® MathXL® PRACTICE

Exercise numbers in green have complete video solutions in MyMathLab.

Problems **1–42** *are the* Quick ✓s *that follow the* EXAMPLES.

Building Skills

In Problems 43–58, find the sum. See Objective 1.

43. $8 + 7$ **44.** $6 + 4$

45. $-5 + 9$ **46.** $-4 + 12$

47. $8 + (-12)$ **48.** $7 + (-13)$

49. $-11 + (-8)$ **50.** $-13 + (-5)$

51. $-16 + 37$ **52.** $-32 + 49$

53. $-119 + (-209)$ **54.** $-145 + (-68)$

55. $-14 + 21 + (-18)$

56. $(-13) + 37 + (-22)$

57. $74 + (-13) + (-23) + 5$

58. $-34 + 46 + (-12) + 72$

In Problems 59–62, determine the additive inverse of each real number. See Objective 2.

59. -325 **60.** -34

61. 125 **62.** 7

In Problems 63–78, find the difference. See Objective 3.

63. $23 - 12$ **64.** $35 - 23$

65. $9 - 17$ **66.** $12 - 19$

67. $-20 - 8$ **68.** $-15 - 9$

69. $13 - (-41)$ **70.** $14 - (-18)$

71. $-36 - (-36)$ **72.** $-15 - (-15)$

73. $0 - 41$ **74.** $0 - 18$

75. $-93 - (-62)$ **76.** $46 - (-25)$

77. $86 - (-86)$ **78.** $49 - (-49)$

In Problems 79–94, find the product. See Objective 4.

79. $5 \cdot 8$ **80.** $7 \cdot 9$

81. $8(-7)$ **82.** $9(-7)$

83. $(0)(-21)$ **84.** $-21 \cdot 0$

85. $(-48)(-3)$ **86.** $(-22)(-5)$

87. $(-42)3$ **88.** $(-128)7$

89. $-5 \cdot 6 \cdot 3$ **90.** $-6 \cdot 4 \cdot 8$

91. $-10(3)(-7)$ **92.** $-8(2)(-9)$

93. $(-2)(4)(-1)(3)(5)$

94. $(-3)(-4)(6)(-1)$

In Problems 95–100, find the multiplicative inverse (or reciprocal) of each number. See Objective 5.

95. 8 **96.** 10 **97.** -4

98. -3 **99.** 1 **100.** 2

In Problems 101–112, find the quotient. See Objective 5.

101. $10 \div 2$ **102.** $36 \div 9$ **103.** $\frac{-56}{-8}$

104. $\frac{-63}{-7}$ 105. $\frac{-45}{3}$ **106.** $\frac{-144}{6}$

107. $\frac{35}{10}$ **108.** $\frac{20}{16}$ **109.** $\frac{60}{-42}$

110. $\frac{120}{-66}$ **111.** $\frac{-105}{-12}$ **112.** $\frac{-80}{-12}$

Mixed Practice

In Problems 113–128, evaluate the expression.

113. $-4 \cdot 18$ **114.** $7 \cdot (-15)$

115. $-16 - (-76)$ **116.** $87 - 19$

117. $-9 \cdot (-19)$ **118.** $7 \cdot 209$

119. $\frac{120}{-8}$ **120.** $\frac{-156}{-26}$

121. $-98 + 56$ **122.** $103 + (-66)$

123. $\frac{75}{|-20|}$ **124.** $\frac{|-42|}{12}$

125. $|-14| + |-26|$ **126.** $|-10| + (-62)$

127. $|-389| - 627$ **128.** $|-193| - (-20)$

In Problems 129–136, write each expression using mathematical symbols. Then evaluate the expression.

129. the sum of 28 and -21

130. the sum of 32 and -64

131. -21 minus 47

132. -85 subtracted from -16

133. -12 multiplied by 18

134. 32 multiplied by -8

135. -36 divided by -108 **136.** -40 divided by 100

Applying the Concepts

In Problems 137–142, write the positive or negative number for each amount or measurement.

137. Stock The price of IBM stock fell by 3.25 dollars.

138. Temperature The current temperature in Juneau, Alaska, is 14° below zero.

139. Chargers Football The Chargers lost 6 yards on the play.

140. Profit Mark's auto dealership showed a profit of \$125,000 this quarter.

141. Checking Account Leila's checking account is now overdrawn by \$48.

142. Census The number of people in Brian's hometown grew by 12,368.

143. Hiking Loren and Richard went on a hiking trip. They walked 8 miles to the base of Snow Creek Falls, where they set up camp. They went another 3 miles to see the Falls and then returned to their campsite. How many miles did they walk that day?

144. Football The Cary High School Eagles took over possession of the ball on their own 15-yard line. The following plays occurred: QB sack, loss of 7 yards; Jon Anderson ran for 14 yards; Juan Ramirez caught a pass for 26 yards. What yard marker is the ball on now?

145. Bank Balance When Martha balanced her checkbook, she had \$563 in the account. Then the following transactions occurred: She wrote a check to Home Depot for \$46, deposited \$233, wrote a check to Vons for \$63, and wrote a check to Petco for \$32. What is Martha's new balance?

146. Checkbook Balance Josie began the month with \$399 in her bank account. She deposited her paycheck of \$839. She paid her \$69 telephone bill, the electric bill for \$78, and rent of \$739. How much does she have left for spending money?

147. Warehouse Inventory The warehouse began the month with 725 cases of soda. During the month the following transactions occurred: 120 cases were shipped out, 590 cases were shipped out, and 310 cases were delivered to the warehouse. Does the warehouse have enough stock on hand to fill an order for 450 cases of soda? What is the difference between what it has and what has been requested?

148. Altitude of an Airplane A pilot leveled off his airplane at 35,000 feet at the beginning of the flight. The following adjustments were made during the trip: gained 4290 feet, dropped 10,400 feet, and then dropped 2605 feet. At what altitude is the plane currently flying?

149. Distance An airplane flying at 25,350 feet is directly over a submarine that is 375 feet below sea level. What is the distance between a person in the plane and a person in the submarine?

150. Elevation The highest point in California is Mt. Whitney at an elevation of 14,495 feet, and the lowest point is in Death Valley at 280 feet below sea level. What is the maximum difference in elevation between the two points in California?

Extending the Concepts

151. Find two integers whose sum is -8 and whose product is 15.

152. Find two integers whose sum is 2 and whose product is -24.

153. Find two integers whose sum is -10 and whose product is -24.

154. Find two integers whose sum is -18 and whose product is 45.

155. The Fibonacci Sequence The numbers in the Fibonacci sequence are $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots$, where each term after the second term is the sum of the two preceding terms. This famous sequence of numbers can be used to model many phenomena in nature.

(a) Form fractions of consecutive terms in the sequence. Find the decimal approximations to $\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}$, and so on.

(b) What number do the ratios get close to? This number is called the **golden ratio** and has application in many different areas.

(c) Research Fibonacci numbers and cite three different applications.

Explaining the Concepts

156. Write a sentence or two that justifies the fact that the product of a positive number and a negative number is a negative number. You may use an example.

157. Explain how $42 \div 4$ may be written as a multiplication problem.

1.5 Adding, Subtracting, Multiplying, and Dividing Rational Numbers

Objectives

1. Multiply Rational Numbers in Fractional Form
2. Divide Rational Numbers in Fractional Form
3. Add or Subtract Rational Numbers in Fractional Form
4. Add, Subtract, Multiply, or Divide Rational Numbers in Decimal Form

Are You Ready for This Section?

Before getting started, take this readiness quiz. If you get a problem wrong, go back to the section cited and review the material.

R1. Find the least common denominator of $\frac{5}{12}$ and $\frac{3}{16}$. [Section 1.2, pp. 11–12]

R2. Rewrite $\frac{4}{5}$ as an equivalent fraction with a denominator of 30. [Section 1.2, pp. 10–11]

R3. Write each rational number in lowest terms. [Section 1.4, pp. 34–35]

(a) $-\frac{18}{30}$ **(b)** $-\frac{24}{4}$

Now that we are comfortable with operations on integers, we can perform operations on rational numbers. We begin with operations on rational numbers expressed as fractions and end the section with the operations on rational numbers in decimal form.

All of the properties included for integers in Section 1.4 apply to rational numbers as well. In fact, these properties apply to all real numbers.

In Words

To find the product of two or more fractions, multiply the numerators together. Then multiply the denominators together. Write the fraction in lowest terms, if necessary.

1 Multiply Rational Numbers in Fractional Form

We use the following property to multiply two rational numbers in fractional form:

Multiplying Fractions

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \quad \text{where } b \text{ and } d \neq 0$$

The rules of signs that apply to integers also apply to rational numbers: The product of two positive rational numbers is positive; the product of a positive rational number and a negative rational number is negative; and the product of two negative rational numbers is positive.

Ready?...Answers **R1.** LCD = 48 **R2.** $\frac{4}{5} = \frac{24}{30}$ **R3.** a. $-\frac{3}{5}$ b. -6

EXAMPLE 1 **Multiplying Rational Numbers (Fractions)**

Find the product: $\frac{2}{9}\cdot\left(-\frac{15}{19}\right)$

Solution

We begin by rewriting the rational number $-\frac{15}{19}$ as $\frac{-15}{19}$. Then we multiply the numerators and multiply the denominators.

$$\frac{2}{9}\cdot\left(\frac{-15}{19}\right) = \frac{2\cdot(-15)}{9\cdot 19}$$

Write the numerator and the denominator as products of prime factors: $= \frac{2\cdot 3\cdot(-5)}{3\cdot 3\cdot 19}$

Divide out common factors: $= \frac{2\cdot \cancel{3}\cdot(-5)}{\cancel{3}\cdot 3\cdot 19}$

$= \frac{2\cdot(-5)}{3\cdot 19}$

Multiply: $= -\frac{10}{57}$

Work Smart

Notice that we do not multiply in the numerator or denominator until we divide out common factors.

Quick ✓

In Problems 1–5, find each product, and write in lowest terms.

1. $\frac{3}{4}\cdot\frac{9}{8}$
2. $\frac{-5}{7}\cdot\frac{56}{15}$
3. $\frac{12}{45}\cdot\left(-\frac{18}{20}\right)$
4. $-\frac{25}{75}\cdot\left(-\frac{9}{4}\right)$
5. $\frac{7}{3}\cdot\frac{1}{14}\cdot\left(-\frac{9}{11}\right)$

2 Divide Rational Numbers in Fractional Form

To divide rational numbers, we must know how to find the reciprocal of a rational number. In Section 1.4, we saw that two numbers are *reciprocals,* or multiplicative inverses, if their product is 1. This definition applies to any nonzero real number. Thus, $\frac{3}{2}$ and $\frac{2}{3}$ are reciprocals because $\frac{3}{2}\cdot\frac{2}{3} = 1$; 9 and $\frac{1}{9}$ are reciprocals because $9\cdot\frac{1}{9} = 1$; $-\frac{4}{7}$ and $-\frac{7}{4}$ are reciprocals because $-\frac{4}{7}\cdot\left(-\frac{7}{4}\right) = 1$.

Quick ✓

6. Two numbers are called multiplicative inverses, or reciprocals, if their product is equal to _.

In Problems 7–10, find the reciprocal of each number.

7. 12
8. $\frac{7}{5}$
9. $-\frac{1}{4}$
10. $-\frac{31}{20}$

We divide rational numbers by rewriting the division as an equivalent multiplication problem.

Dividing Rational Numbers Expressed as Fractions

$$\frac{a}{b}\div\frac{c}{d} = \frac{a}{b}\cdot\frac{d}{c} = \frac{a\cdot d}{b\cdot c} \quad \text{where } b, c, d \neq 0$$